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Development of a stepwise primary diagnosis method of a beam using a force identification and optimization approach

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Abstract

In this study, a stepwise diagnosis method is proposed. In this method, the location of abnormality is first estimated using the force identification approach. The point of the proposed method is that the abnormality is considered as an additional local force in the early stage of abnormality. After that, the cause of abnormality is identified. That is, the response of the structure with certain abnormality at the estimated location is calculated, the residual between the actual and calculated response is checked, and the correct cause of abnormality is identified. As a numerical example, a uniform beam supported by two springs is considered. Four types of abnormality are assumed to occur and the validity and applicability of the proposed method is checked. As a result, using the data without measurement error, the location and cause of abnormality can be exactly identified. On the other hand, using the data with measurement error, the location of abnormality can be estimated in the case of accurate measurement, and the cause of abnormality can be identified with sufficient accuracy. Therefore, the diagnosis method proposed in this paper is considered to be useful for the primary diagnosis.

1. Introduction

Machine condition monitoring and diagnosis has become increasingly important, and the application of these techniques to beam structures and rotating machinery has been widely investigated. At the early stage of diagnosis, abnormal data is often encountered, and a primary diagnosis is sometimes necessary to identify the location and cause of abnormality. There are many researches on diagnosis of a beam with a crack. Some of recent researches are shown. Reng et al. [1] proposed a nonlinear frequency-response functions to detect cracks. They treated a breathing crack and showed the higher harmonics in output responses. Al-Said [2] identified a crack in a beam based on the change of natural frequencies. Guo and Billings [3] proposed a detection method of cracks in a beam using the coupled map lattice that is a discrete computation method for complex systems. Xiang et al. [4] and Loutridis et al. [5] used the wavelet analysis to detect a crack in a beam. One of interesting approaches is the use of antiresonance. Wahl et al. [6], Bamnios et al. [7], Dilena and Morassi [8] and Dharmaraju and Sinha [9] proposed a diagnosis method of a crack using antiresonance.

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The crack is one of the most important abnormalities, but other ones may happen on a structure, for example, lack or additional mass. In a primary diagnosis, the cause, location and extent of abnormality hope to be identified. To do this, an optimization approach is often used. The abnormal response is calculated from a mathematical model having a certain cause of abnormality, and the residual between the actual and calculated response is checked and the correct cause of abnormality can then be identified as the cause leading to a minimum residual. However, this approach is not always useful, because an exact mathematical model of the abnormality is difficult to construct, and the measurement error has influence on the residual.

In the field of rotating machinery, a model-based fault diagnosis has been proposed by Sekhar [10]. In this method, the response of the whole system is expressed as the superposition of vibration modes, whose number is less than the number of sensors, and it is estimated using the measured responses. Then, the external force is identified by inserting the responses estimated into the equation of motion. And the change of response by the abnormality is considered due to an additional external force, and the correct cause of abnormality is identified by the optimization technique such that the additional external force will coincide with the resultant force by abnormality. However, there is a slight amount of error between the estimated and correct response. Due to this error, in the case of a uniform structure such as a beam, the external force identified is distributed on the structure even if the external force is an impulsive one [11], while in the case of a rotor system, the unbalance force is identified at the rotor position, so the force identification method is useful.

In this study, a stepwise diagnosis method is proposed. First, the location of the abnormality is estimated using the force identification approach based on the transfer function. The point of the proposed method is that the abnormality is considered as an additional force on a finite element in the early stage of abnormality. In this way, the distributed external force in the force identification process can be avoided. After that, the cause of abnormality is identified. That is, the response of the structure with a certain abnormality at the estimated location is calculated, the residual between the actual and calculated response is checked, and the correct cause of abnormality is identified. As a numerical example, a uniform beam supported by two springs is considered. Four types of abnormality are considered, i.e., a crack, change of Young's modulus, an additional mass and change of density. Firstly, the validity of the proposed method is checked using the data without error, and then the applicability is checked using the data with error.

2. Diagnosis method using a force identification approach

2.1. Construction of mathematical model

For purposes of simplicity, the structure to be diagnosed is assumed to be without damping, and a vibration test will be performed at regular intervals to monitor the health of the structure. The mass matrix $[M]$ and the stiffness matrix $[K]$ of the structure under the normal condition have been constructed in advance by using FEM. The equation of motion is expressed as follows:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{f\}. \tag{1}$$

The external force $\{f\}$ for the vibration test can be measured and the response $\{x\}$ is measured at several points, so the output equation is

$$\{x_m\} = [C]\{x\}, \tag{2}$$

where $\{x_m\}$ is the response vector measured and $[C]$ is the coefficient matrix.

A harmonic excitation with frequency ω is considered as the external force

$$\{f\} = \{F\}e^{j\omega t} \tag{3}$$

and the responses are expressed as follows:

$$\{x\} = \{X\}e^{j\omega t}, \quad \{x_m\} = \{X_m\}e^{j\omega t}. \tag{4}$$

The equation of motion and the output equation are transformed into the following:

$$([K] - \omega^2[M])\{X\} = \{F\}, \quad \{X_m\} = [C]\{X\}. \tag{5}$$

2.2. Estimation of the location of abnormality

An abnormality is determined to exist in the structure when the response changes significantly compared with the normal one in the vibration test. After such a determination, the diagnosis will proceed.

In this study, a stepwise diagnosis method is proposed. First, the location of abnormality is estimated using the force identification approach, then the cause of abnormality is identified.

In the abnormal case, the response changes because the mass and/or stiffness matrices change due to abnormality even if the same external force acts on the structure in the vibration test. However, the change of response is considered to be the result of an additional external force $\{f_a\} = \{F_a\}e^{j\omega t}$ exerted on the normal structure as follows:

$$([K] - \omega^2[M])\{X_a\} = \{F\} + \{F_a\}, \tag{6}$$

where

$$\{X_a\} = \{X\} + \{\Delta X\}, \quad \{\Delta X_m\} = [C]\{\Delta X\}. \tag{7}$$

Moreover, an abnormality in the early stage occurs locally in the structure, so the abnormality is expressed as an additional impulsive force; that is, an impulsive force acts on the ith element of the finite element model. When the structure is modeled using a beam element by FEM, the translational forces f_{xi} and f_{xj}, and the rotational forces $f_{\theta i}$ and $f_{\theta j}$ act on both ends of the element, but no external force acts on other elements. Here, when the responses are measured at four points ($m = 4$), the four force elements can be identified. The external force that acts on the ith element is expressed as

$$\{F_{ai}\} = \left\{ f_{xi} \quad f_{\theta i} \quad f_{xj} \quad f_{\theta j} \right\}^{\mathrm{T}}. \tag{8}$$

It is noted that this external force is not actual one, but virtual one.

The four force elements can be identified using the relationship

$$\{\Delta X_m\} = [H(\omega)]\{F_{ai}\}, \tag{9}$$

where $[H(\omega)]$ is a transfer function between the responses measured and the external force assumed. In the actual practice of solving Eq. (9), the transfer function $[H(\omega)]$ is often ill-posed, and thus the truncation of small singular values is needed. By the singular value decomposition, the transfer matrix $[H(\omega)]$ is expressed as follows:

$$[H(\omega)] = [U][B][V]^{\mathrm{T}}, \tag{10}$$

where $[U]$ and $[V]$ are 4×4 matrices, respectively, and $[B]$ is a diagonal matrix whose elements are singular values σ_i ($i = 1,\ldots,4$). For example, when the rank of $[B]$ is 3, that is, σ_4 is significantly small, $\{F_{ai}\}$ is identified as follows:

$$\{F_{ai}\} = [V]\left[\tilde{B}\right]^{-1}[U]^{\mathrm{T}}\{\Delta X_m\}, \tag{11}$$

where

$$\left[\tilde{B}\right]^{-1} = \mathrm{diag}\left[1/\sigma_1 \quad 1/\sigma_2 \quad 1/\sigma_3 \quad 0 \right]. \tag{12}$$

By using the $\{F_{ai}\}$ identified in Eq. (11), the responses are calculated and $\{\Delta X_{mi}\}$ is obtained as follows:

$$\{\Delta X_{mi}\} = [H(\omega)][V]\left[\tilde{B}\right]^{-1}[U]^{\mathrm{T}}\{\Delta X_m\} = [U][B]\left[\tilde{B}\right]^{-1}[U]^{\mathrm{T}}\{\Delta X_m\}. \tag{13}$$

The next objective function J_1 is calculated for every element number i:

$$J_1 = |\{\Delta X_m\} - \{\Delta X_{mi}\}|, \tag{14}$$

where $|\cdot|$ means the Euclid norm and the abnormality will occur at the element where the objective function J_1 is significantly small.

2.3. Identification of the cause of abnormality

A mathematical model with the jth cause of abnormality in the ith element is constructed. The mass and stiffness matrices are expressed as $[M_{ai}^{(j)}]$ and $[K_{ai}^{(j)}]$, respectively. When the external force for the vibration test acts on the mathematical model, the response $\{X_{ai}^{(j)}\}$ is calculated using the equation of motion

$$\left(\left[K_{ai}^{(j)}\right] - \omega^2\left[M_{ai}^{(j)}\right]\right)\left\{X_{ai}^{(j)}\right\} = \{F\}, \tag{15}$$

and the response at the measurement points are obtained as follows:

$$\left\{X_{aim}^{(j)}\right\} = [C]\left\{X_{ai}^{(j)}\right\}. \tag{16}$$

The next objective function J_2 is calculated as

$$J_2 = \left|\{X_m\} - \left\{X_{aim}^{(j)}\right\}\right|, \tag{17}$$

and the cause of abnormality is identified when the objective function J_2 is significantly small.

3. Numerical examples

3.1. Model for diagnosis

Fig. 1 shows the model to be diagnosed, which consists of a beam supported by two springs. To construct a mathematical model, the beam is divided into 20 elements and the characteristic matrices are obtained by FEM using a beam element. The spring properties are added to the stiffness matrix corresponding to the node 2 and 20. The characteristics of the model are as follows:

- Length of beam, 1.0 m; width of beam, 0.1 m; thickness of beam, 0.016 m;
- Young's modulus of beam, 210 GPa; density of beam, $7.85 \times 10^3\,\mathrm{kg\,m^{-3}}$;
- Spring constant at node number 2, $30.0\,\mathrm{kN\,m^{-1}}$; spring constant at node number 20, $28.0\,\mathrm{kN\,m^{-1}}$.

These values are the identified ones of experimental equipment under construction.

The lower four natural frequencies are 10.60, 16.84, 86.70 and 234.86 Hz. The external force for the vibration test acts on node 11, whose amplitude and frequency are 10.0 N and 6.0 Hz, respectively. The responses at the nodes 3, 8, 13 and 18 are measured.

3.2. Assumed abnormalities

Though many kinds of abnormality may occur on the structure, four types of abnormality are assumed to occur at the 6th element in this study: (1) a crack, (2) change of Young's modulus, (3) an additional mass and

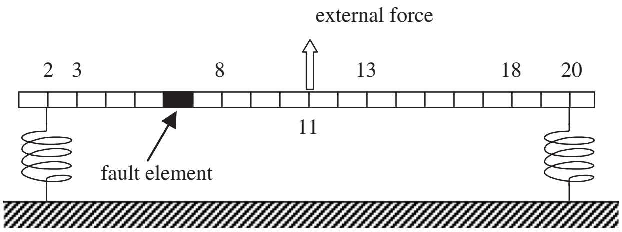

Fig. 1. Model of structure.

(4) change of density. To normalize the extent of abnormality, the residual of response in the case of a crack is considered as a reference value, and the extents of other abnormalities will be determined as follows:

- *Crack*: An actual crack in the early stage is usually considered as a breathing crack and thus the characteristic nonlinear vibration occurs, but in this study, it is considered as an open crack. Though many types of open crack model have been proposed, the model proposed by Sinha et al. [12] is used in this study. The stiffness matrix is modified so as to reduce its rigidity, in which the location ($\alpha = l_c/l_e$) and depth ($\beta = t_c/t_e$) as shown in Fig. 2(a) are variables. The element stiffness matrix of the cracked element is concretely written as
$$[K_{\text{crack}}] = [K] - [K_c], \tag{18}$$
where $[K]$ is the element stiffness matrix without crack and $[K_c]$ is the reduction in the stiffness due to a crack and a function of α and β. As an example of an abnormality, the location α is set to be 0.25 and depth β is 0.15, 0.30 or 0.50, respectively. For example, when the depth is set to be 0.15, the residual of responses at four measurement points $|\{\Delta X_m\}|$ is obtained as 6.3382×10^{-7} m. So the extent of each abnormality is adapted so as to agree with the residual of 6.3382×10^{-7} m. For other cases of depth, the extents of abnormalities are adapted.
- *Change of Young's modulus*: The Young's modulus of the 6th element is multiplied by 0.800232 to make the residual and the reference value agree.
- *Additional mass*: The mass matrix with additional mass is sum of the one of uniform beam and the one of additional mass, which is obtained from the kinetic energy
$$T = \tfrac{1}{2} m \dot{u}(l_m, t)^2, \tag{19}$$
where m is additional mass, $\dot{u}$ is the velocity and l_m is the location of additional mass shown in Fig. 2(b). The mass of 3.042×10^{-2} kg at a quarter point ($\alpha = 0.25$) from the left end on the 6th element additionally is attached to make the residual and the reference value agree.
- *Change of density*: The density of the 6th element is multiplied by 1.04828 to make the residual and the reference value agree.

The modification factors of abnormalities are summarized in Table 1. And the exact response and the abnormal one are shown in Fig. 3 when the depth β is equivalent to 0.50.

3.3. Estimation of the location of abnormality using the date without measurement error

The validity of the proposed method is checked using the data without measurement error. A local force is identified using the abnormal data of four kinds of abnormality, respectively, based on the procedures

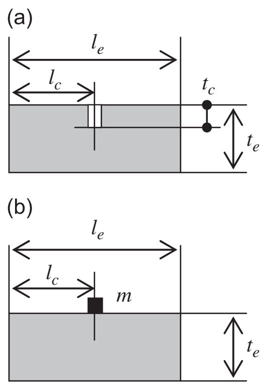

Fig. 2. Special finite element: (a) element with crack and (b) element with additional mass.

Table 1
Analysis conditions to obtain the abnormal responses

Crack (location $\alpha = 0.25$; depth β)	Decreased Young's modulus (ratio to normal value)	Additional mass (location $\alpha = 0.25$; mass, kg)	Increased density (ratio to normal value)	Residual (m)
0.15	0.800232	3.04225d−2	1.048282	0.63382d−06
0.30	0.593480	8.33296d−2	1.132253	0.17402d−05
0.50	0.339016	0.2359854	1.374574	0.49621d−05

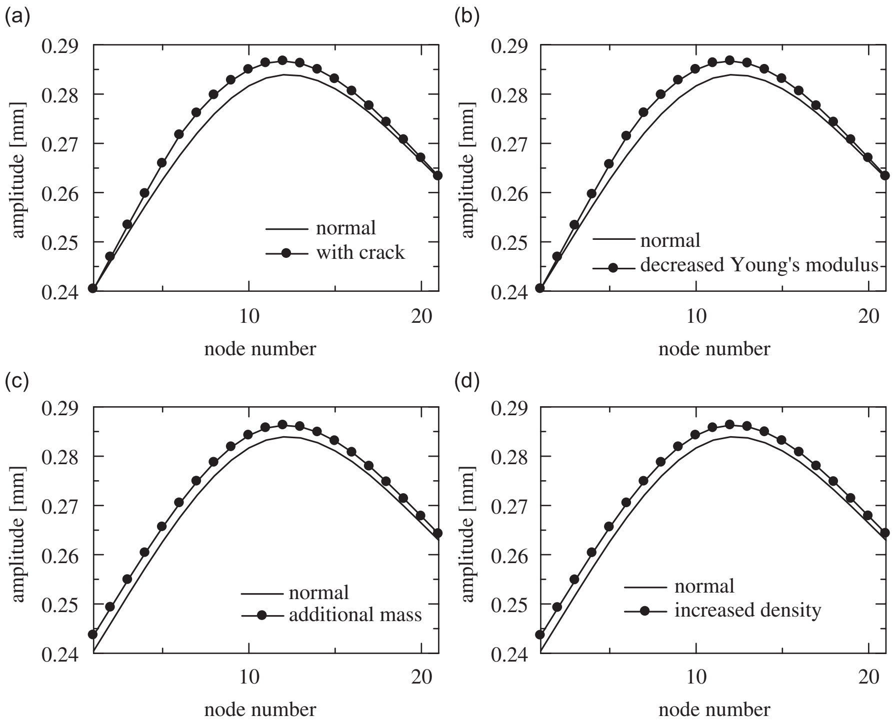

Fig. 3. Comparison of response between normal and abnormal condition: (a) the case of crack, (b) the case of decreased Young's modulus, (c) the case of additional mass and (d) the case of increased density.

described in Chapter 2.2. The objective function J_1 of Eq. (14), which is the residual, is shown in Fig. 4. As shown from the figure, the residuals at the 5th, 6th and 7th elements are significantly small. This fact indicates that an abnormality may occur at this location. And it is shown that the changes of J_1 of Figs. 4(a) and (b) are similar because the crack model used in this paper is the one in which the Young's modulus is modified corresponding to the depth of crack, and the ones of Figs. 4(c) and (d) are similar because the additional mass and the change of density have effects on the inertia.

3.4. Identification of the cause of abnormality using the date without measurement error

In the diagnosis method proposed in this paper, a cause of abnormality is sought only for the 5th, 6th or 7th element. At first, however, the cause of abnormality is checked for all elements, and the results of the diagnosis are discussed. There are four possible causes of the abnormal data, and each is handled differently:

In the case of a crack, the element number, the location in the element (at intervals constituting 1% of the length from 1% to 100%) and the depth of the crack (at intervals constituting 1% of the thickness from 1% to 99%) are set as variables and the residual is checked.

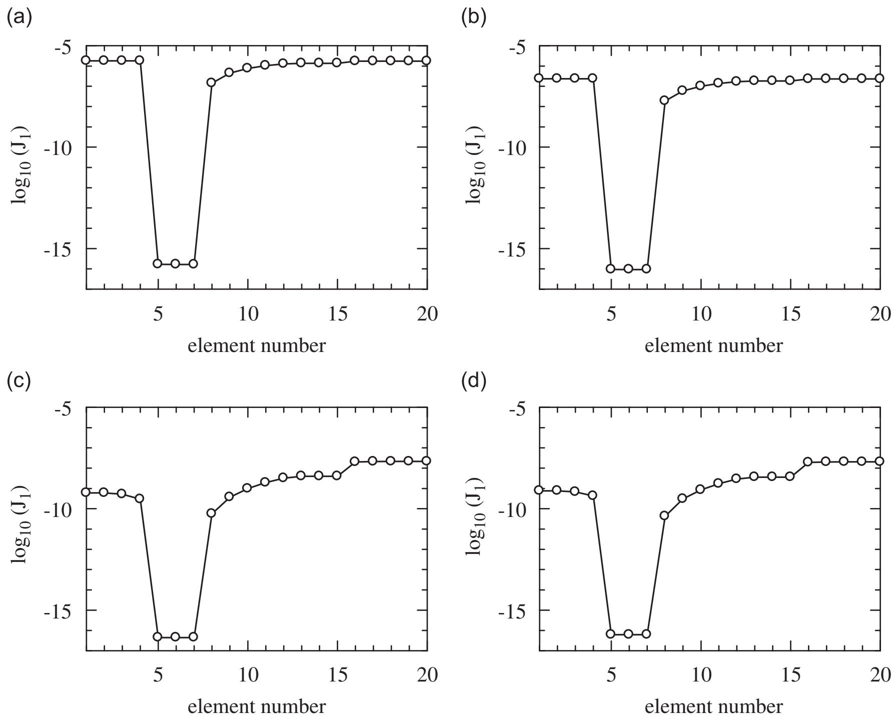

Fig. 4. Objective function J_1 in the case of using four kinds of abnormal condition data without error: (a) the case of using crack data, (b) the case of using decreased Young's modulus data, (c) the case of using additional mass data and (d) the case of using increased density data.

In the case of decreased Young's modulus, the element number and the degree of decrease (at intervals constituting 1% of Young's modulus from 1% to 99%) are set as variables and the residual is checked.

In the case of an additional mass, the element number, the location in the element (at intervals constituting 1% of the length from 1% to 100%) and the magnitude of mass (at intervals of 1×10^{-4} kg from 1×10^{-4} to 0.25 kg) are set as variables and the residual is checked.

In the case of increased density, the element number and the degree of increase (at intervals constituting 1% of density from 101% to 199%) are set as variables and the residual is checked.

The results are summarized in Figs. 5 and 6. In each figure, the horizontal axis denotes the element number and the vertical axis shows the minimum residual J_2 of Eq. (17) in each element. It is noted that the condition of minimum residual is important, not the minimum value of residual, because the minimum value may be reduced by using smaller interval.

Fig. 5(a) is the result when the correct cause of abnormality is crack and the assumed cause of abnormality is also crack. The other hand, Fig. 5(b) is the one when the correct cause of abnormality is crack but the assumed cause of abnormality is the decrease of Young's modulus. The conditions of minimum residual are shown in Table 2. From Fig. 5 or Table 2, when the correct cause of abnormality is crack, the cause cannot be identified, that is, the cause may be crack or decrease of Young's modulus in element 6, or additional mass or increase of density in element 8. The estimation result of the location of abnormality indicates that the abnormality may occur at the 5th, 6th or 7th element so that the cause of abnormality can be identified as crack or decrease of Young's modulus. However, it is difficult to distinguish between the crack and the decrease of Young's modulus because of the reason shown in Section 3.3. When the correct cause of abnormality is the decrease of Young's modulus, the similar results can be obtained as the case of crack.

Fig. 6 is the result when the correct cause of abnormality is additional mass. In the case, the cause cannot be identified, that is, the cause may be crack or decrease of Young's modulus in element 3, or additional mass or increase of density in element 6. The estimation result of the location of abnormality indicates that the abnormality may occur at the 5th, 6th or 7th element so that the cause of abnormality can be identified as

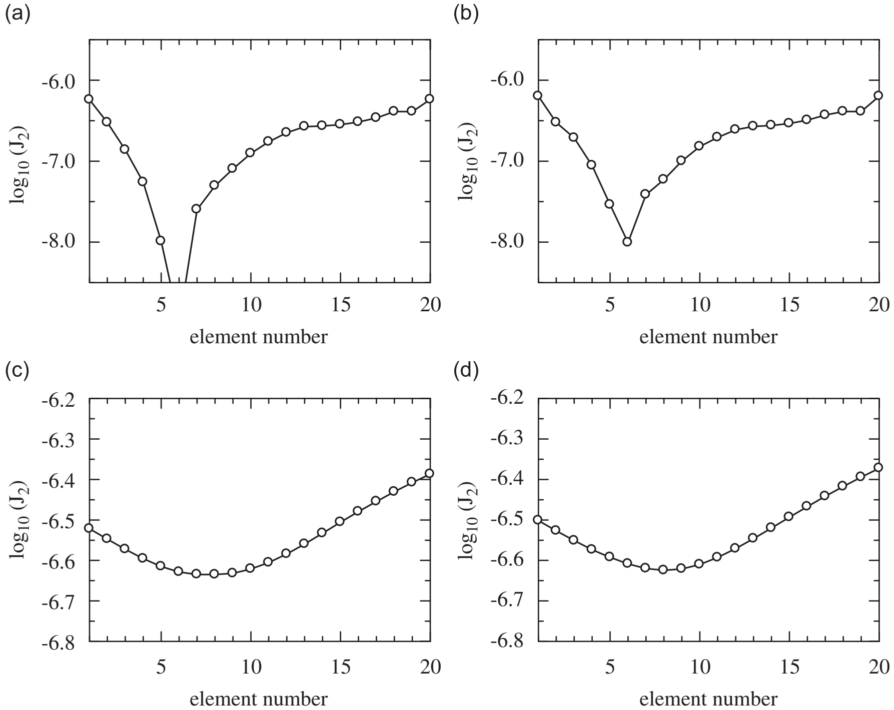

Fig. 5. Objective function J_2 using data without error when the correct cause is crack: (a) the case on the assumption of crack, (b) the case on the assumption of decreased Young's modulus, (c) the case on the assumption of additional mass and (d) the case on the assumption of increased density.

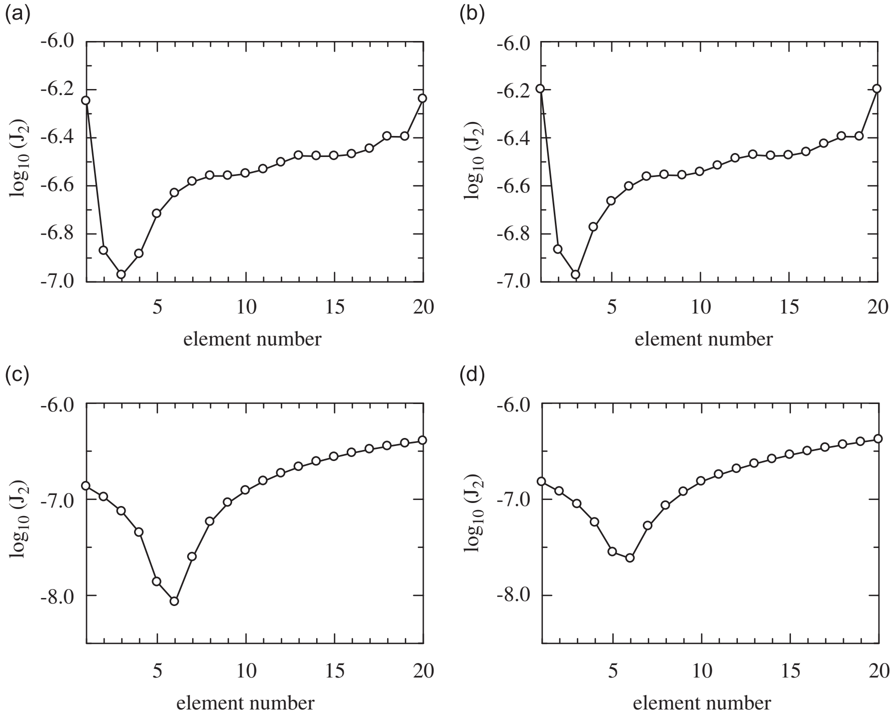

Fig. 6. Objective function J_2 using data without error when the correct cause is additional mass: (a) the case on the assumption of crack, (b) the case on the assumption of decreased Young's modulus, (c) the case on the assumption of additional mass and (d) the case on the assumption of increased density.

Table 2
Results of diagnosis without measurement error

Correct causes of abnormality	Assumed causes of abnormality			
	Crack	Decreased Young's modulus	Additional mass	Increased density
Crack	Element no. 6 Location 0.25 Depth 0.15	Element no. 6 Ratio 0.80	Element no. 8 Location 0.26 Mass 0.0276	Element no. 8 Ratio 1.04
Decreased Young's modulus	Element no. 6 Location 0.55 Depth 0.16	Element no. 6 Ratio 0.80	Element no. 8 Location 0.44 Mass 0.0274	Element no. 8 Ratio 1.04
Additional mass	Element no. 3 Location 0.29 Depth 0.79	Element no. 3 Ratio 0.38	Element no. 6 Location 0.25 Mass 0.0304	Element no. 6 Ratio 1.05
Increased density	Element no. 3 Location 0.29 Depth 0.78	Element no. 3 Ratio 0.38	Element no. 6 Location 0.50 Mass 0.0303	Element no. 6 Ratio 1.05

additional mass or increase of density. But the causes cannot be distinguished because they have same effect on the dynamic behavior of the system. When the correct cause of abnormality is the increase of density, the similar results can be obtained as the case of additional mass.

3.5. *Estimation of the location of abnormality using the date with measurement error*

The applicability of the proposed method is checked using the data with measurement error. The approximate number to three, five or seven significant digits is considered as the measured data. The location of abnormality is estimated through additional force identification so that the difference of response between normal and abnormal condition must be measured with high accuracy.

The objective function J_1 is shown in Fig. 7 when the correct cause of abnormality is crack. In Fig. 7(a) of the slightest abnormality case, there is no difference between normal and abnormal data in the case of three significant digits. From the figures, it is shown that when the extent of abnormality becomes large, the location can be identified even if the accuracy of measured date decreases. It is recognized that the accuracy of measured data must be five significant digits in this case.

When the correct cause of abnormality is decrease of Young's modulus, the similar results can be obtained as Fig. 7.

Fig. 8 shows the result in the case of additional mass. Comparing with the results of crack, the minimum value of J_1 isn't distinct a little in the case of five significant digits. And it is recognized that the accuracy of measured data must be also five significant digits in these cases.

When the correct cause of abnormality is increase of density, the similar results can be obtained as Fig. 8.

3.6. *Identification of the cause of abnormality using the date with measurement error*

In this section, the data in the case of five significant digits and equivalent to the crack data of 30% crack depth is used as the measured data.

The results are summarized in Figs. 9 and 10 and the conditions of minimum residual are shown in Table 3 as same as Figs. 5 and 6 and Table 2. As similar as the case without measurement error, when the correct cause of abnormality is crack or decrease of Young's modulus, the cause cannot be identified only by the minimum residual J_2, but it can be identified as crack or decrease of Young's modulus under the constraint of location. When the correct cause of abnormality is additional mass or increase of density, the cause can be identified under the constraint of location.

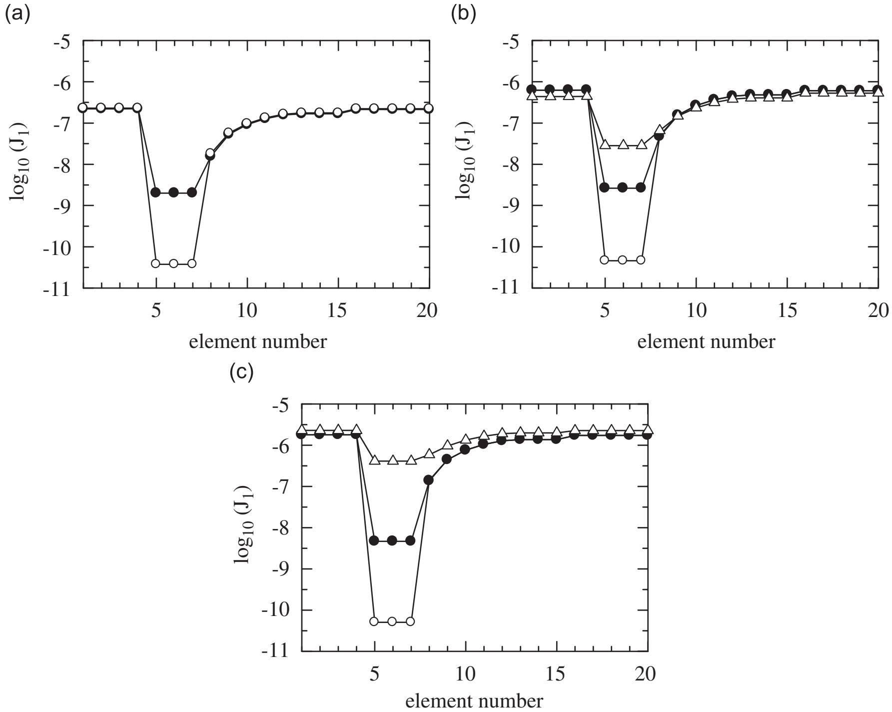

Fig. 7. Objective function J_1 in the case of using crack data with error (○: significant digit 7; ●: significant digit 5; △: significant digit 3): (a) the case of crack depth 0.15, (b) the case of crack depth 0.30 and (c) the case of crack depth 0.50.

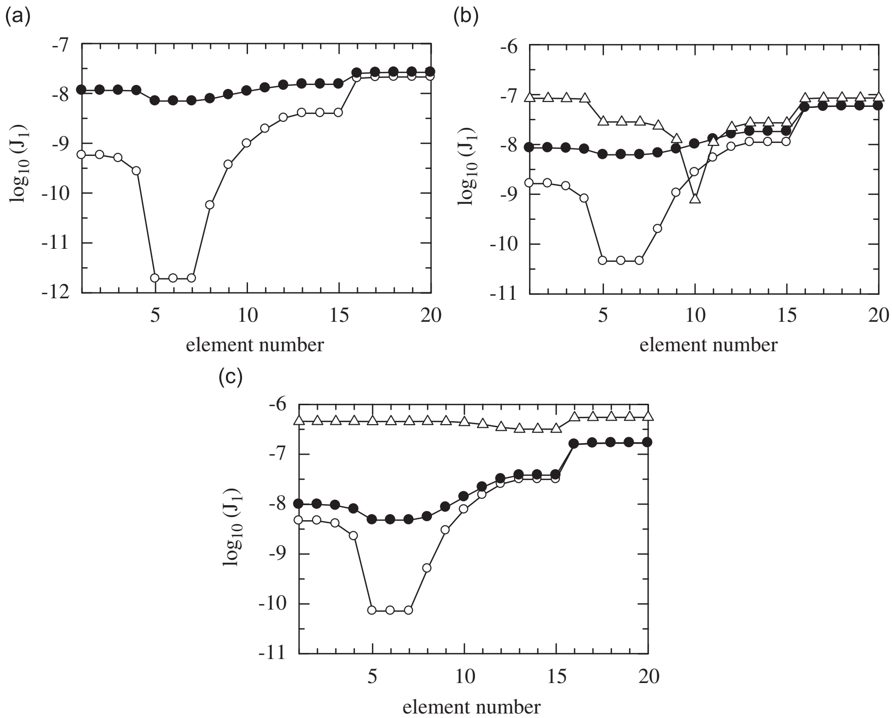

Fig. 8. Objective function J_1 in the case of using additional mass data with error. (○: significant digit 7; ●: significant digit 5; △: significant digit 3): (a) the case equivalent to crack depth 0.15, (b) the case equivalent to crack depth 0.30 and (c) the case equivalent to crack depth 0.50.

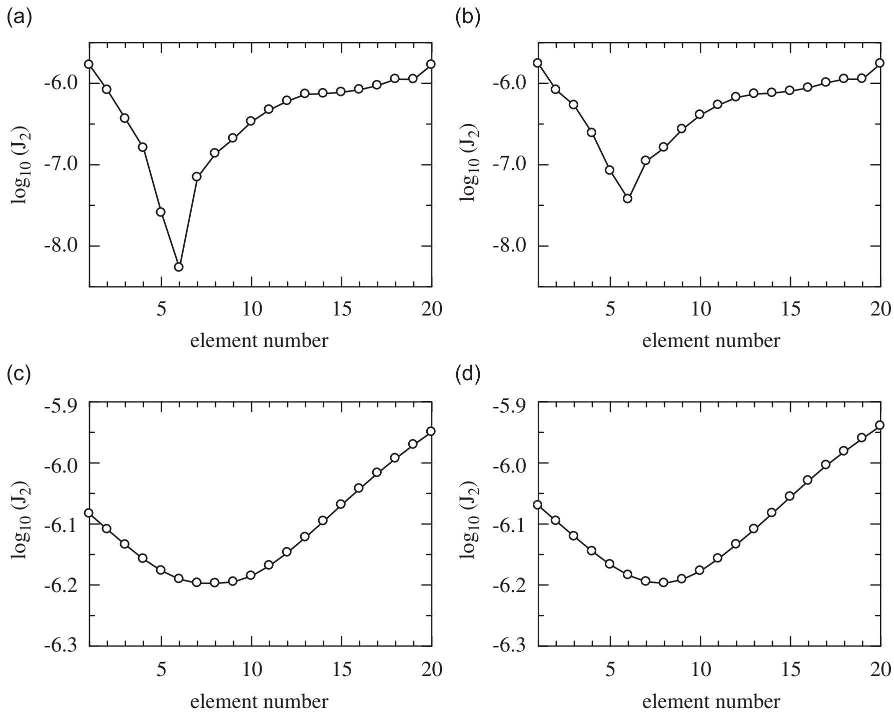

Fig. 9. Objective function J_2 using data with error when the correct cause is crack: (a) the case on the assumption of crack, (b) the case on the assumption of decreased Young's modulus, (c) the case on the assumption of additional mass and (d) the case on the assumption of increased density.

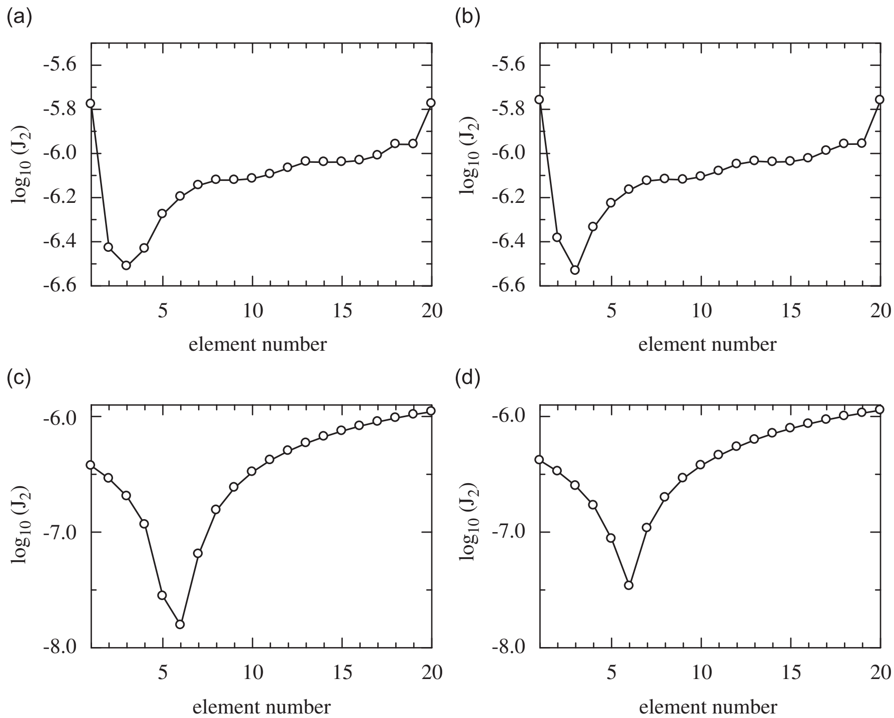

Fig. 10. Objective function J_2 using data with error when the correct cause is additional mass: (a) the case on the assumption of crack, (b) the case on the assumption of decreased Young's modulus, (c) the case on the assumption of additional mass and (d) the case on the assumption of increased density.

Table 3
Results of diagnosis with measurement error

Correct causes of abnormality	Assumed causes of abnormality			
	Crack	Decreased Young's modulus	Additional mass	Increased density
Crack	Element no. 6 Location 0.25 Depth 0.30	Element no. 6 Ratio 0.59	Element no. 8 Location 0.28 Mass 0.0756	Element no. 8 Ratio 1.12
Decreased Young's modulus	Element no. 6 Location 0.49 Depth 0.47	Element no. 6 Ratio 0.59	Element no. 8 Location 0.43 Mass 0.0754	Element no. 8 Ratio 1.12
Additional mass	Element no. 3 Location 0.25 Depth 0.78	Element no. 3 Ratio 0.18	Element no. 6 Location 0.30 Mass 0.0832	Element no. 6 Ratio 1.13
Increased density	Element no. 3 Location 0.25 Depth 0.78	Element no. 3 Ratio 0.18	Element no. 6 Location 0.48 Mass 0.0831	Element no. 6 Ratio 1.13

The extent of abnormality can be identified accurately even if the measured data has error except the case of additional mass. In that case, it is expected that the identified location is 0.25 and the mass is 0.833×10^{-2} kg because the correct location is 0.25 and the correct mass is 0.833296×10^{-2} kg. But the identified results are the location is 0.3 and the extent of 0.832×10^{-2} kg. In this study, the minimum value of J_2 was searched by the method described in Section 3.4. It is expected that the accurate result can be obtained by the use of optimization approach.

4. Conclusions

In this study, a stepwise diagnosis method was proposed. In this method, the location of abnormality is first estimated using the force identification approach. The point of the proposed method is that the abnormality is considered as an additional local force in the early stage of abnormality. After that, the cause of abnormality is identified. As a numerical example, a uniform beam supported by two springs is considered. Four types of abnormality, that are a crack, change of Young's modulus, an additional mass and change of density, are assumed to occur and the validity and applicability of the proposed method is checked. As a result, using the data without measurement error, the location of abnormality can be exactly estimated and the cause of abnormality can be also identified, so the proposed method is recognized to be valid. Next, using the data with measurement error, the location of abnormality can be estimated in the case of accurate measurement because the difference of response between normal and abnormal condition is important. And the cause of abnormality can be identified with sufficient accuracy. Therefore the diagnosis method proposed in this paper is considered to be useful for the primary diagnosis.

Applying this method to actual diagnosis, however, some problems may arise such as to construct the accurate mathematical model is difficult. Therefore, the experimental verifications are necessary. Now the experiment is under preparation and the verification results will be the forthcoming studies.

Acknowledgments

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